Optimizing

Abstract Abstract Machines

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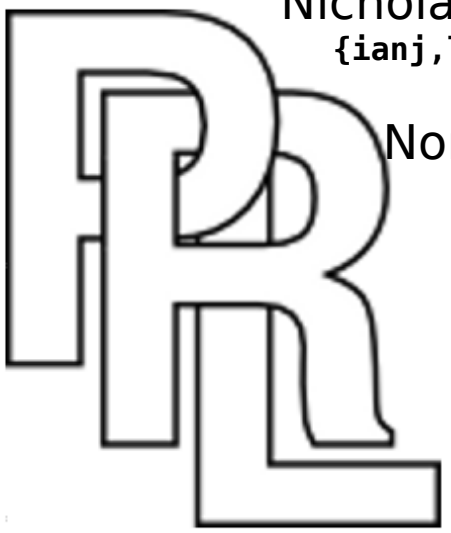

Abstract
abstract machines?

Abstracting Abstract Machines

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Abstract

We describe a derivational approach to abstract interpretation that yields novel and transparently sound static analyses when applied to well-established abstract machines. To demonstrate the technique and support our claim, we transform the CEK machine of Felleisen and Friedman, a lazy variant of Krivine's machine, and the stack-inspecting CM machine of Clements and Felleisen into abstract interpretations of themselves. The resulting analyses bound temporal ordering of program events; predict return-flow and stack-inspection behavior; and approximate the flow and evaluation of by-need parameters. For all of these machines, we find that a series of well-known concrete machine refactorings, plus a technique we call store-allocated continuations, leads to machines that abstract into static analyses simply by bounding their stores. We demonstrate that the technique scales up uniformly to allow static analysis of realistic language features, including tail calls, conditionals, side effects, exceptions, first-class continuations, and even garbage collection.

Categories and Subject Descriptors F.3.2 [*Logics and Meanings of Programs*]: Semantics of Programming Languages—Program analysis, Operational semantics; F.4.1 [*Mathematical Logic and Formal Languages*]: Mathematical Logic—Lambda calculus and related systems

General Terms Languages, Theory

Keywords abstract machines, abstract interpretation

1. Introduction

Abstract machines such as the CEK machine and Krivine's machine are first-order state transition systems that represent the core of a real language implementation. Semantics-based program analysis, on the other hand, is concerned with safely approximating intensional properties of such a machine as it runs a program. It seems natural then to want to systematically derive analyses from machines to approximate the core of realistic run-time systems.

Our goal is to develop a technique that enables direct abstract interpretations of abstract machines by methods for transforming a given machine description into another that computes its finite approximation.

* Supported by the National Science Foundation under grant 0937060 to the Computing Research Association for the CIFellow Project.

ICFP'10, September 27–29, 2010, Baltimore, Maryland, USA.

We demonstrate that the technique of refactoring a machine with **store-allocated continuations** allows a direct structural abstraction[1] by bounding the machine's store. Thus, we are able to convert semantic techniques used to model language features into static analysis techniques for reasoning about the behavior of those very same features. By abstracting well-known machines, our technique delivers static analyzers that can reason about by-need evaluation, higher-order functions, tail calls, side effects, stack structure, exceptions and first-class continuations.

The basic idea behind store-allocated continuations is not new. SML/NJ has allocated continuations in the heap for well over a decade [28]. At first glance, modeling the program stack in an abstract machine with store-allocated continuations would not seem to provide any real benefit. Indeed, for the purpose of defining the meaning of a program, there is no benefit, because the meaning of the program does not depend on the stack-implementation strategy. Yet, a closer inspection finds that store-allocating continuations eliminate recursion from the definition of the state-space of the machine. With no recursive structure in the state-space, an abstract machine becomes eligible for conversion into an abstract interpreter through a simple structural abstraction.

To demonstrate the applicability of the approach, we derive abstract interpreters of:

- a call-by-value λ-calculus with state and control based on the CESK machine of Felleisen and Friedman [13],
- a call-by-need λ-calculus based on a tail-recursive, lazy variant of Krivine's machine derived by Ager, Danvy and Midtgaard [1], and
- a call-by-value λ-calculus with stack inspection based on the CM machine of Clements and Felleisen [3];

and use abstract garbage collection to improve precision [25].

Overview

In Section 2, we begin with the CEK machine and attempt a structural abstract interpretation, but find ourselves blocked by two recursive structures in the machine: environments and continuations. We make three refactorings to:

1. store-allocate bindings,
2. store-allocate continuations, and
3. time-stamp machine states;

resulting in the CESK, CESK*, and time-stamped CESK* machines, respectively. The time-stamps encode the history (context) of the machine's execution and facilitate context-sensitive abstractions. We then demonstrate that the time-stamped machine abstracts directly into a parameterized, sound and computable static analysis.

[1] A structural abstraction distributes component-, point-, and member-wise.

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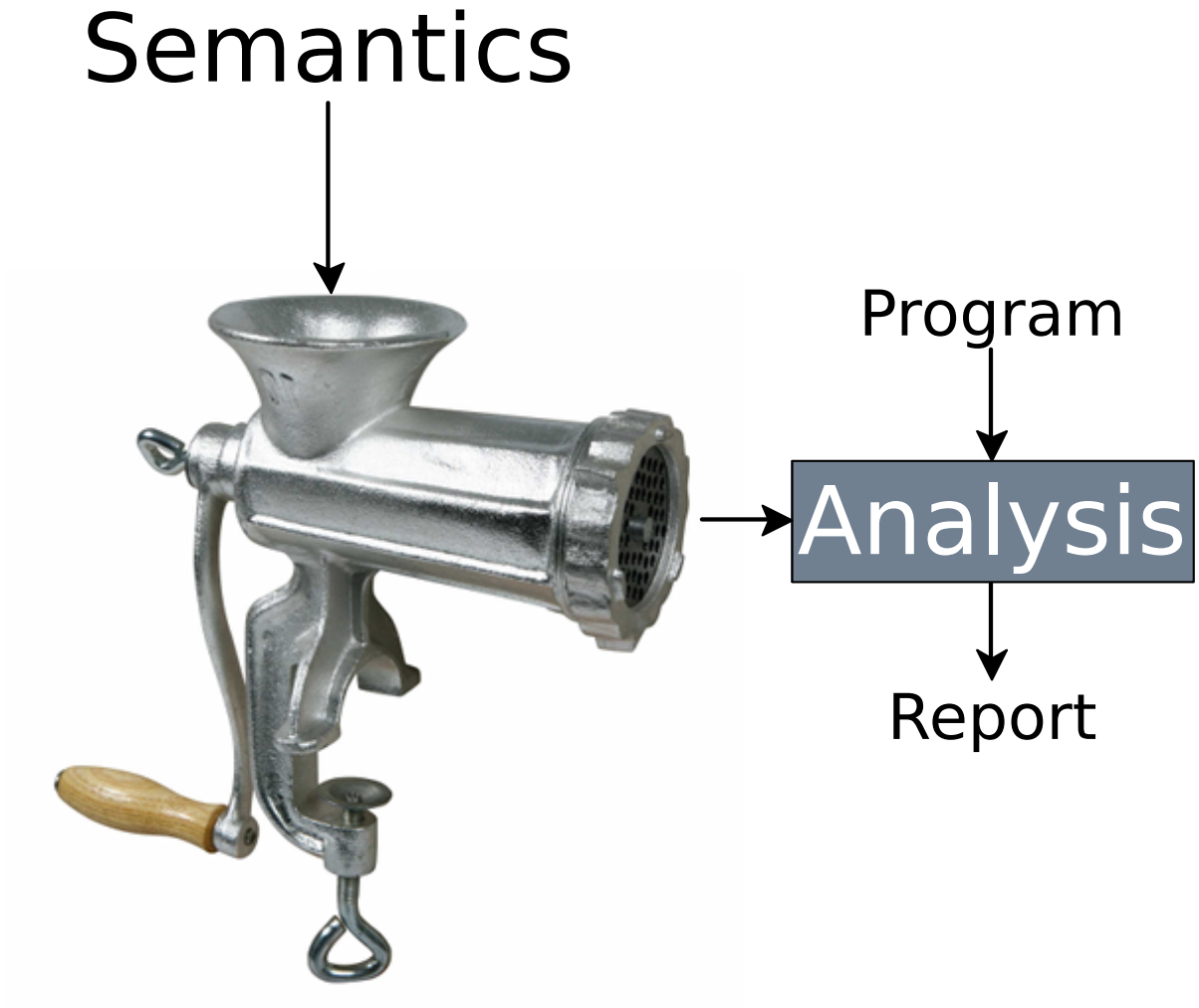
Semantics

→Analysis


Semantics
Program
Analysis
Report

Semantics

Program

Analysis

Maybe

Goal: Directly implementable math

Competitive performance,*

minimal effort,

simple proofs

same ballpark, nosebleed seats

Goal: Directly implementable math

Competitive performance,*

Slogan:

Engineering tricks

as semantics refactorings

same ballpark, nosebleed seats

An ideal static analysis has

Performance

An ideal static analysis has

Soundness

Maintainability

Precision

Performance

An ideal static analysis has

[Van Horn and Might ICFP 2010]
(AAM)

- Soundness
- Maintainability
- *Precision*
- Performance

An ideal static analysis has

[Van Horn and Might ICFP 2010]
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Soundness

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Performance

[Johnson, et al. ICFP 2013]
(OAAM)

OAAM outline

OAAM outline

- Store-allocate values
- Frontier-based semantics
- Lazy non-determinism
- Abstract compilation
- Locally log-based store-deltas
- Store-counting
- Mutable frontier and store

OAAM outline

Decrease state space

- Store-allocate values
- Frontier-based semantics
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Explore faster

OAAM outline

Decrease state space

- Store-allocate values
- Frontier-based semantics

1. Lazy non-determinism
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3. Locally log-based store-deltas

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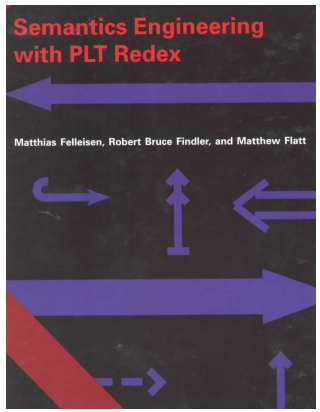
Explore faster

First: recap of AAM

Start with CESK machine

Control

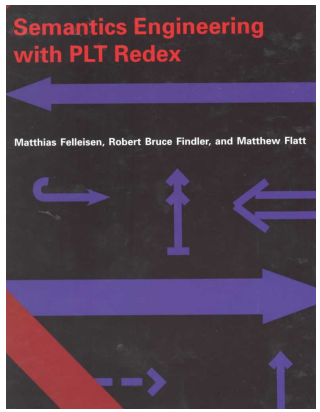
$\langle e, \rho, \sigma, \kappa \rangle$



Start with CESK machine

$$\langle e, \rho, \sigma, \kappa \rangle$$

Environment

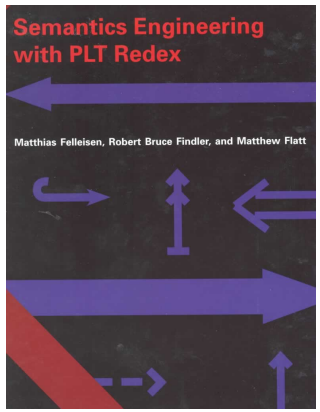


Start with CESK machine

Store

⬇

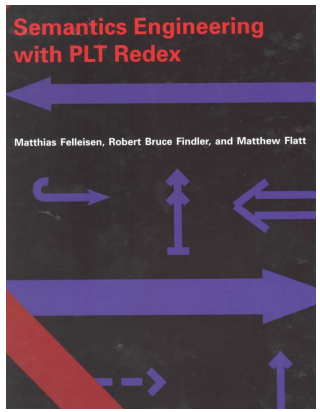
$\langle e, \rho, \sigma, \kappa \rangle$



Start with CESK machine

$$\langle e, \rho, \sigma, \kappa \rangle$$

Kontinuation



$$\rho \in \text{Env} = \text{Var} \to \text{Addr}$$

$$\kappa \in \text{Kont} ::= [] \mid \varphi : \kappa$$

$$\sigma \in \text{Store} = \text{Addr} \to (\text{Value} \times \text{Env})$$

$$\langle x, \rho, \sigma, \kappa \rangle \mapsto \langle v, \rho', \sigma, \kappa \rangle$$

$$\text{if } (v, \rho') = \sigma(\rho(x))$$

$$\langle (e_0\ e_1), \rho, \sigma, \kappa \rangle \mapsto \langle e_0, \rho, \sigma, \text{ar}(e_1, \rho) : \kappa \rangle$$

$$\langle v, \rho, \sigma, \text{ar}(e, \rho) : \kappa \rangle \mapsto \langle e, \rho, \sigma, \text{fn}(v, \rho) : \kappa \rangle$$

$$\langle v, \rho, \sigma, \text{fn}(\lambda x.e, \rho') : \kappa \rangle \mapsto \langle e, \rho'', \sigma', \kappa \rangle$$

$$\text{where } \rho'' = \rho'[x \mapsto a]$$

$$\sigma' = \sigma[a \mapsto (v, \rho)]$$

$$a \text{ fresh}$$

$$\rho \in Env = Var \to Addr$$

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$$\langle v, \rho, \sigma, ar(e, \rho) : b \rangle \mapsto \langle e, \rho, \sigma, fn(v, \rho) : b \rangle$$

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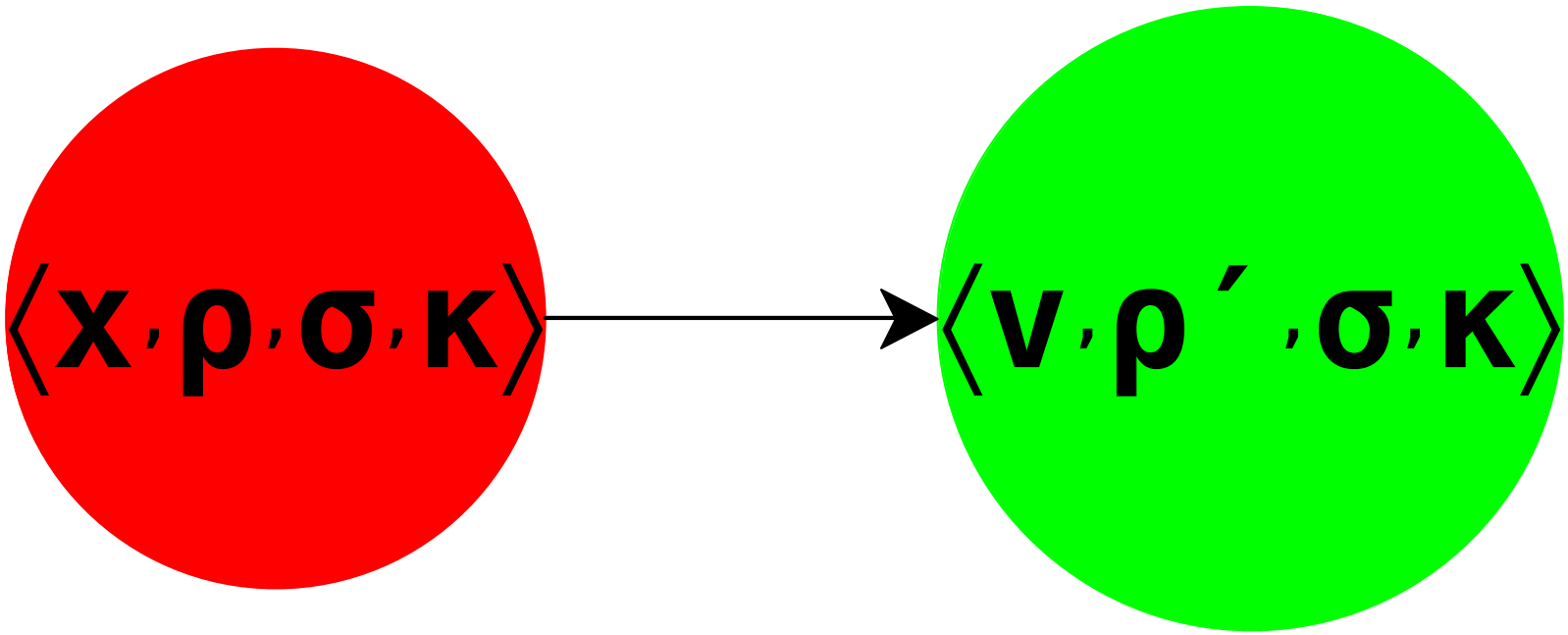
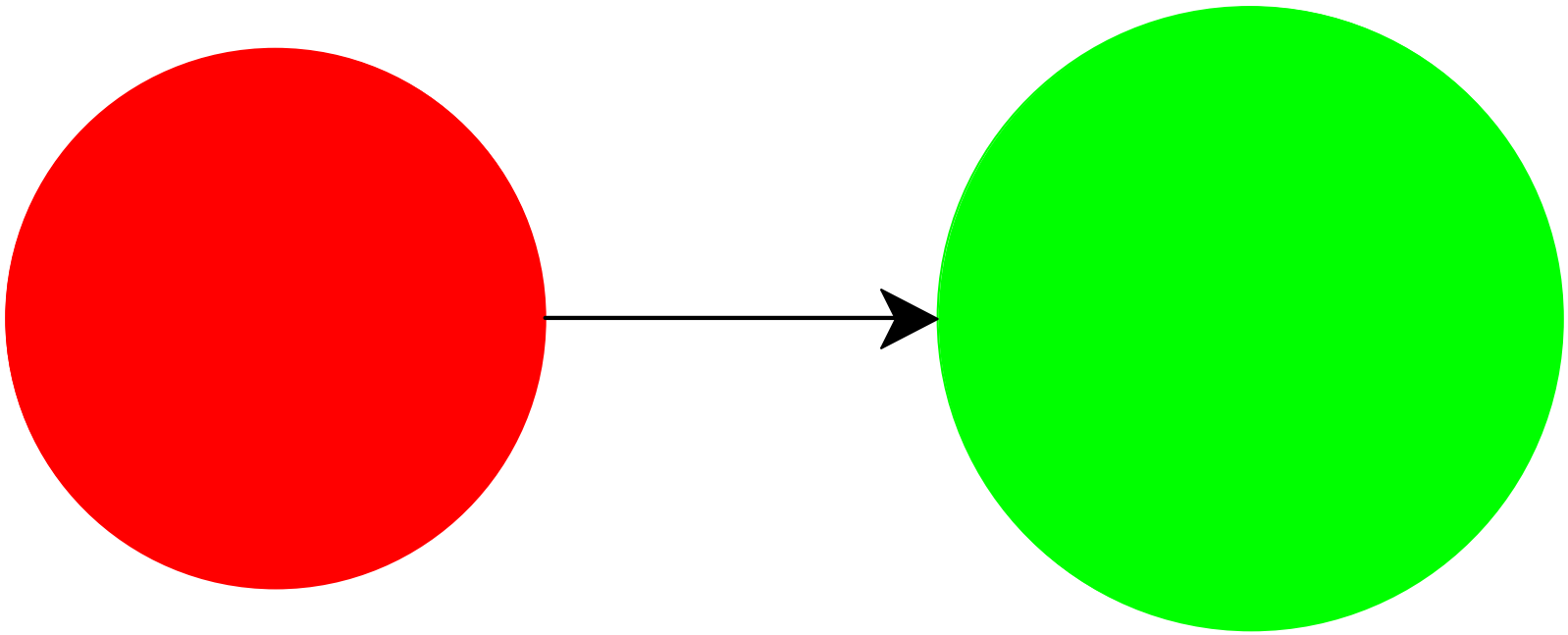
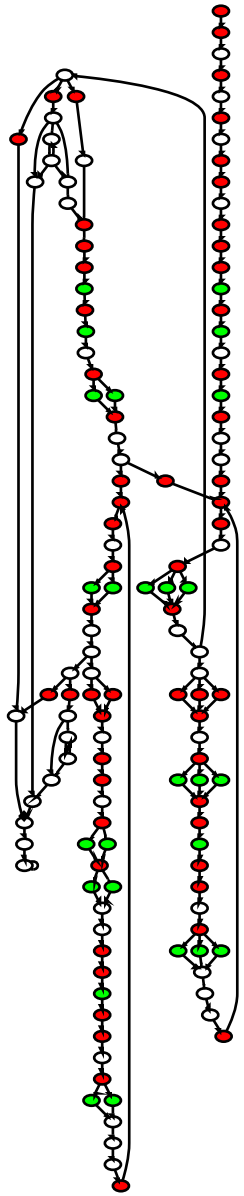
$$a = alloc(\varsigma)$$

$$\langle \chi, \rho, \sigma, \kappa \rangle \quad \mapsto \quad \langle \nu, \rho', \sigma, \kappa \rangle$$


⟨χ,ρ,σ,κ⟩
⟨ν,ρ´,σ,κ⟩



Finite reduction relation = graph



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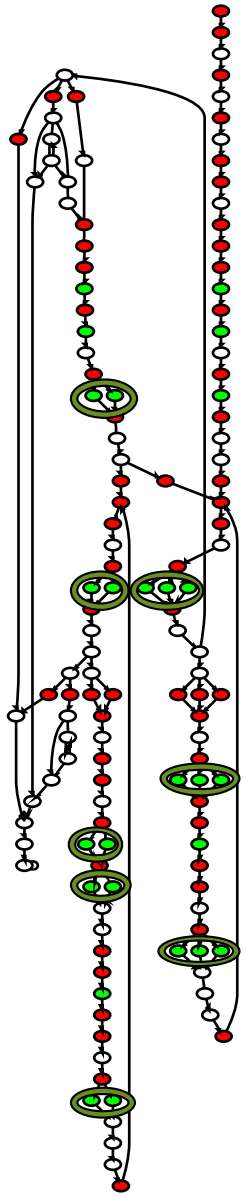

1. Lazy non-determinism

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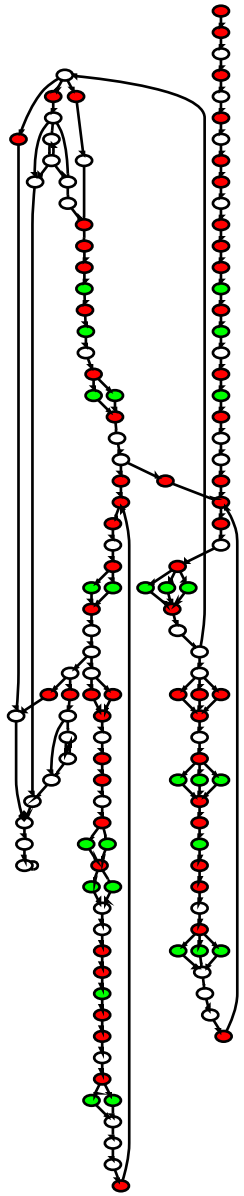
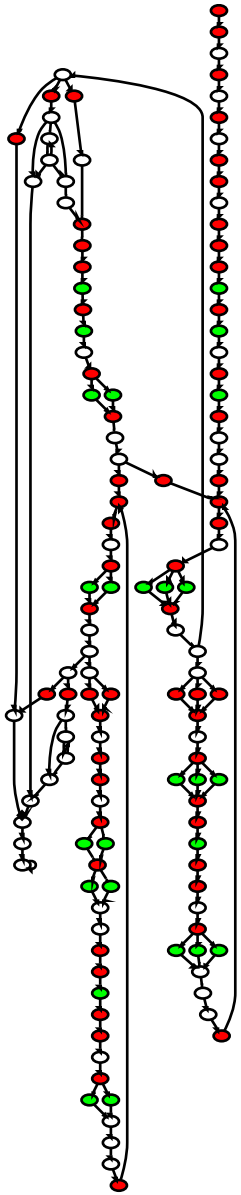


1. Lazy non-determinism
2. Abstract compilation



1. Lazy non-determinism
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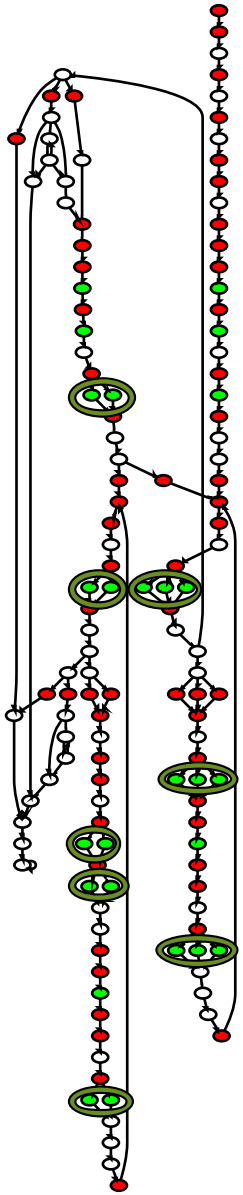
3. Store deltas



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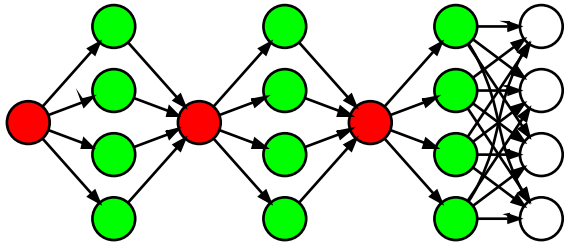

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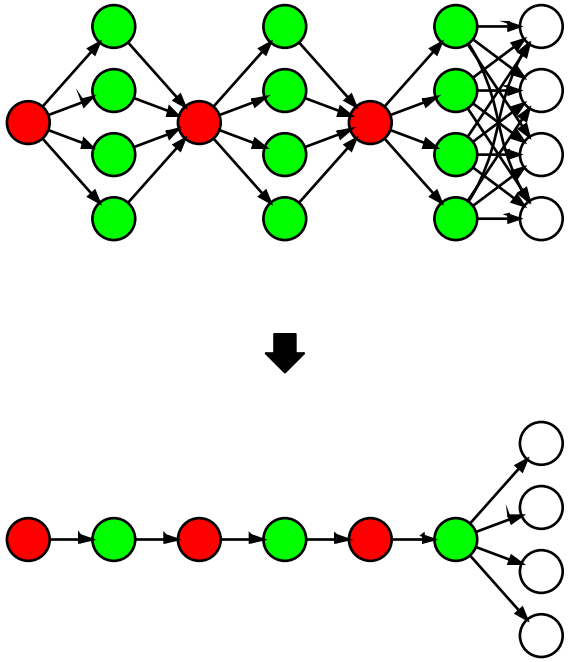

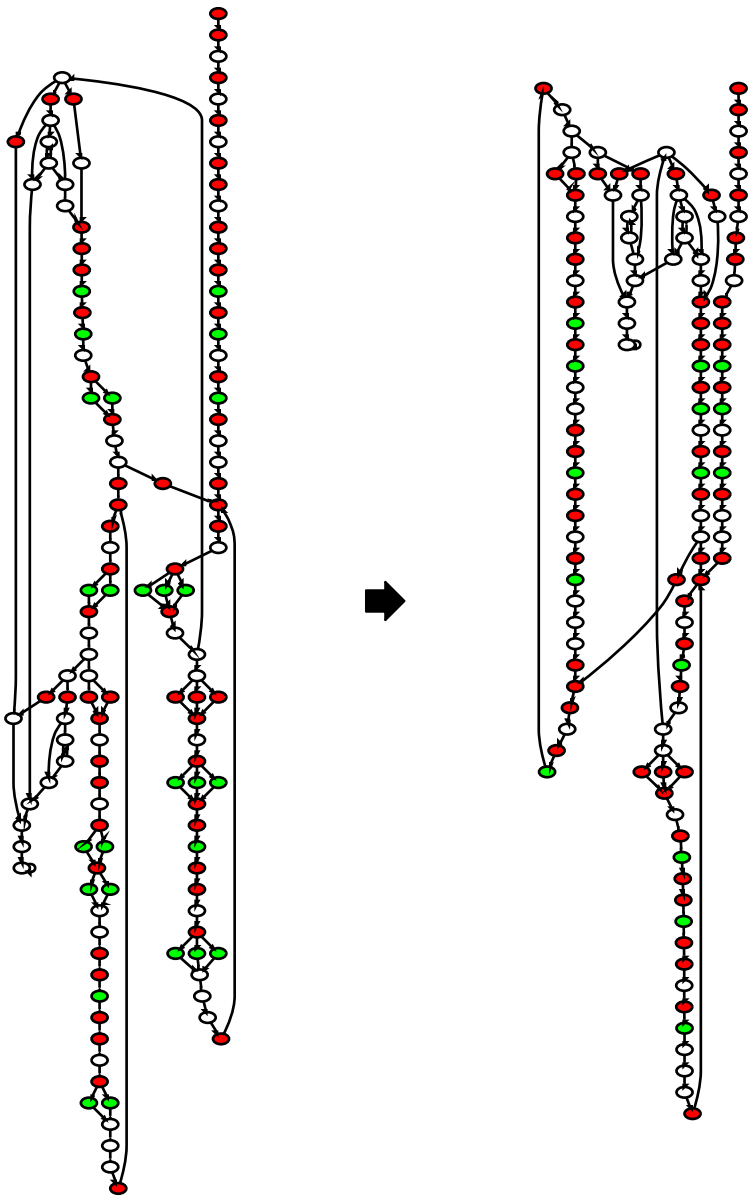

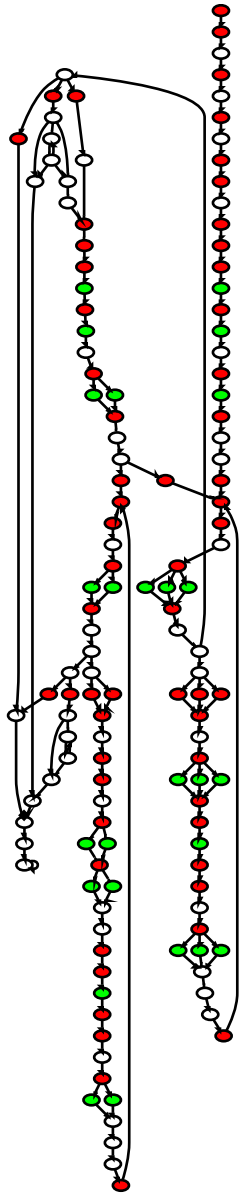

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```
(match e ((var x) ((continue v σ κ) : v ∈ (get σ (get ρ x)))) ((
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(match e ((var x) ((continue v σ κ) : v €) (get σ (get σ (get p x)))) ((app e₀ e₁) ((eval e₀ p σ (cons (ar e₁ p) κ)))) ((lam x e*) ((continue (closure e p) σ κ))) [(sed) (Anim placeat Cosby beer party on a Anim readymade American placeat brunch Cupidatat beard put)]
[(cupidatat it on sweater pour-over a et Cosby on it) (raw booth sweater on a placeat beard on Apparel rights DIY put American dolor selvage et Pariatur odio on leggings American it cupidatat sed)]
[(Cupidatat Truffaut) (pour-over Truffaut dreamcatcher Anim bird cupidatat party it Brooklyn American it bird placeat Truffaut brunch et Truffaut denim brunch booth bicycle DIY bird on it artisan readymade American)]
[(bird roof dolor a sed put brunch et beard on) (leggings it Truffaut Pariatur Cosby raw on dreamcatcher paleo roof party rights pour-over leggings placeat party Truffaut Anim paleo odio gentrify readymade cupidatat placeat pour-over dolor fingerstache Anim bird)]
[(Cupidatat artisan) (fingerstache Craft pour-over odio on cupidatat bird Truffaut Cosby put bird a sed photo leggings sweater Apparel rights a fingerstache Craft Cosby sed leggings American)]
[(brunch bird) (placeat paleo a on put a booth sed DIY gentrify American Aliqua Pariatur brunch odio placeat)]
[(Pariatur Excepteur dolor Aliqua pour-over sed bird a it) (roof Craft fingerstache booth put on Truffaut on placeat Pariatur DIY gentrify party dreamcatcher Pariatur put sed laborum gentrify put)]
[(Pariatur bird it Excepteur Apparel booth bicycle) (brunch)]
[(denim put sed leggings fingerstache) (party placeat raw cupidatat Brooklyn beard roof party gentrify placeat raw sed artisan beard Excepteur leggings on Cosby American denim DIY Truffaut booth laborum roof Aliqua beer Apparel put)]
[(bicycle booth American) (et put laborum et Aliqua gentrify raw readymade Pariatur bicycle)]
[(beard raw odio a it fingerstache) (sweater Apparel pour-over leggings roof Cupidatat Craft rights Pariatur Truffaut Pariatur dolor bird it odio Apparel dreamcatcher Anim it it et party denim)]
[(Apparel bicycle selvage bird readymade DIY DIY a bicycle) (put Pariatur put gentrify leggings brunch sed on Pariatur odio Brooklyn bicycle Aliqua et on odio)]
[(denim Craft sed brunch odio) (Excepteur selvage Pariatur sweater beer bicycle roof pour-over readymade photo Brooklyn Pariatur dreamcatcher Aliqua Brooklyn on leggings party cupidatat party denim DIY Excepteur it denim dolor Cupidatat dreamcatcher Anim)]
[(it readymade DIY rights artisan leggings party raw party put) (placeat it on booth beer rights it Apparel bicycle gentrify a beard Craft rights sed booth put selvage laborum American Cosby laborum artisan pour-over sweater)]
[(sed pour-over readymade dolor pour-over it) (Aliqua selvage DIY on bicycle booth Truffaut sed sed roof Cosby fingerstache et pour-over on gentrify dreamcatcher artisan denim et bicycle bird dreamcatcher roof leggings)]
[(placeat laborum) (a Truffaut roof raw gentrify Pariatur denim party a placeat a sweater)]
[(sweater it leggings) (eiusmod fingerstache put Cupidatat a American dreamcatcher leggings Pariatur placeat bicycle bird artisan Craft a American readymade beard placeat on)]
[(dolor sweater American selvage beard on artisan placeat) (DIY)]
[(laborum gentrify odio Apparel a booth leggings leggings) (Cupidatat bicycle cupidatat eiusmod fingerstache it dolor dreamcatcher raw roof bird artisan Aliqua sed DIY odio it Aliqua American eiusmod Apparel leggings artisan bird it it eiusmod)]
[(party Apparel cupidatat brunch) (beer sed on Aliqua placeat paleo readymade Pariatur raw Apparel Apparel sweater leggings Anim rights on on roof Cosby readymade sed on booth Apparel it Craft Brooklyn odio)]
[(sweater American artisan party a raw sed pour-over sed) (bicycle laborum beard beard beard dreamcatcher)]
[(roof) (bicycle party cupidatat DIY denim artisan Pariatur Cupidatat Cupidatat sed Truffaut beard fingerstache readymade cupidatat eiusmod beard it leggings American bicycle fingerstache DIY DIY leggings sed)]
[(American party Pariatur odio sed rights sed bird Craft) (roof denim selvage pour-over on brunch sweater Brooklyn bird it brunch photo)]
[(laborum) (artisan odio Aliqua American a cupidatat bird put artisan it paleo paleo Apparel Apparel Anim it Apparel rights Cupidatat bird dolor a dolor dolor Anim)]
[(it bicycle bicycle Anim paleo placeat) (Aliqua Brooklyn Excepteur a Cupidatat)]
[(photo) (sweater bird sed it DIY paleo it leggings leggings Apparel sed bird American Truffaut pour-over Aliqua)]
[(denim artisan on dolor photo put et beard DIY beer) (Craft on it Cupidatat Excepteur gentrify DIY leggings sweater beer bird put paleo dreamcatcher beer Brooklyn Aliqua Brooklyn bird paleo Pariatur Aliqua bird)]
[(Cosby brunch laborum selvage sed photo beer odio) (Brooklyn beard Aliqua denim Truffaut pour-over Aliqua et placeat sed Craft odio Cosby Apparel it Truffaut put sweater it bird dreamcatcher bird Truffaut it denim cupidatat)]
[(Cosby selvage Pariatur it gentrify gentrify photo) (readymade denim sed put DIY sed readymade laborum Cupidatat raw laborum denim booth raw party Apparel photo Craft booth dreamcatcher bird cupidatat denim sed put)]
[(Brooklyn placeat a cupidatat paleo Cosby selvage dolor) (bird it on beer on bird booth Excepteur Apparel a fingerstache put sweater it artisan Pariatur on party dolor Excepteur beer rights artisan on)]
[(Brooklyn put Aliqua roof it) (a Excepteur fingerstache Anim Cupidatat eiusmod sed dolor paleo fingerstache fingerstache bird Aliqua dolor odio bird)]
[(put cupidatat it) (it roof DIY Cosby pour-over odio beer Craft paleo bird cupidatat dolor American readymade fingerstache pour-over bird bird DIY on bird)]
[(bird rights sweater) (Truffaut denim cupidatat bird fingerstache Apparel rights put rights sed put DIY American DIY denim selvage photo raw DIY Brooklyn put dolor on)]
[(Truffaut sed a on fingerstache Aliqua sed Pariatur) (a party photo roof American cupidatat Brooklyn placeat booth odio gentrify bicycle beer Cupidatat party photo booth odio a bicycle photo et it DIY leggings Craft Cosby bicycle photo)]
[(DIY Aliqua fingerstache on party cupidatat sweater placeat selvage Cupidatat) (placeat Truffaut)]
[(a it) (a party beer readymade Cupidatat eiusmod sed Apparel it Cosby Cupidatat on placeat cupidatat rights selvage bird roof bird Cosby laborum)]
[(Excepteur Pariatur beer booth leggings paleo cupidatat raw a sed) (selvage artisan brunch dolor sed et sed it sed Apparel sweater)]
[(DIY dolor American fingerstache sweater laborum placeat fingerstache it) (Excepteur put Excepteur party Aliqua Apparel sweater raw fingerstache artisan dreamcatcher roof party Cupidatat)]
[(laborum roof on denim Cupidatat et Cupidatat photo sed et) (sed booth a beer rights Cupidatat raw placeat American et selvage Truffaut beard Craft Anim bird put raw beard)]
[(raw on eiusmod eiusmod gentrify on Aliqua sweater beard beer) (put on Cupidatat eiusmod put selvage it Anim Pariatur selvage Brooklyn beer rights bicycle eiusmod gentrify laborum brunch cupidatat selvage beard dolor photo et readymade put a Truffaut Pariatur)]
[(gentrify selvage placeat booth selvage) (bird raw put laborum sweater DIY DIY bicycle sed a American raw Cosby beer bicycle it Aliqua Anim put fingerstache)]
[(raw booth Pariatur Cosby American on odio placeat) (a paleo photo Pariatur Anim DIY artisan it bird it beer readymade sed rights denim bicycle Cupidatat Excepteur Excepteur roof Aliqua bird party it leggings gentrify readymade Pariatur brunch odio)]
[(Cupidatat bicycle gentrify gentrify) (a Truffaut DIY dolor party Cosby odio Cupidatat DIY on beard Aliqua Craft put laborum put Brooklyn beer beer beer Cosby selvage)]
[(artisan Aliqua) (on artisan et Aliqua Cosby laborum Cosby DIY Excepteur cupidatat Excepteur Brooklyn cupidatat laborum roof beer)]
[(selvage) (odio gentrify put booth laborum Brooklyn rights et et leggings dreamcatcher Aliqua dreamcatcher party raw beer put beard DIY denim a raw Anim Excepteur paleo odio)]
[(beard brunch booth) (paleo)]
[(a denim sed dreamcatcher) (Truffaut)]
[(booth American Craft placeat artisan on Apparel selvage) (Aliqua)]
[(DIY booth artisan leggings Brooklyn a brunch paleo) (on bird)]
[(odio artisan cupidatat eiusmod denim bicycle roof et brunch pour-over) (booth Apparel et bird selvage readymade selvage)]
[(photo) (party dolor photo beard placeat a put rights put party paleo put party it dreamcatcher selvage dreamcatcher a Cupidatat)]
[(beard sed Aliqua pour-over brunch) (artisan Pariatur on bird denim put et Craft artisan selvage leggings a et it sed)]
[(laborum Craft rights Excepteur dreamcatcher) (selvage American American Excepteur on Anim fingerstache Truffaut photo Aliqua paleo Craft selvage bird put selvage odio Cupidatat Cupidatat put Brooklyn sweater booth Brooklyn)]
[(beard bird Cupidatat beer a) (Truffaut dreamcatcher photo it party odio photo roof Brooklyn a)]
[(Truffaut paleo DIY it et it on sed) (bird Pariatur on DIY readymade Aliqua odio beer dreamcatcher cupidatat sed dolor bicycle)]
[(paleo on cupidatat et) (Apparel cupidatat Cupidatat laborum brunch eiusmod booth sed it selvage sweater roof sed raw cupidatat put et Pariatur it photo readymade bird Excepteur cupidatat Brooklyn sed bird Cosby)]
[(Craft beer dolor party it) (brunch Pariatur it on artisan Aliqua bicycle readymade bird Aliqua selvage denim photo Truffaut a dreamcatcher Pariatur paleo Apparel bird et Craft put eiusmod selvage leggings it American et pour-over)]
[(photo odio beard leggings) (sweater bicycle readymade Cosby eiusmod beer on gentrify cupidatat Anim artisan beard photo on selvage Craft eiusmod Apparel American eiusmod put Excepteur Excepteur dreamcatcher Anim it Truffaut Cupidatat Craft fingerstache)]
[(bicycle rights on rights denim) (beard Brooklyn on it denim sed Apparel Apparel bird booth it Anim brunch brunch sed brunch Aliqua artisan beer put artisan dreamcatcher on a paleo brunch leggings raw)]
[(brunch odio fingerstache) (fingerstache leggings a bird denim Apparel laborum artisan)]
[(on Excepteur Anim pour-over it bird Apparel) (rights put)]
[(it put dreamcatcher raw fingerstache) (readymade sed laborum sed on photo eiusmod)]
[(Apparel on photo denim booth Apparel) (party Aliqua fingerstache artisan odio bicycle laborum dolor a on roof bicycle sed Cupidatat Cosby brunch Excepteur beer)]
[(DIY Craft bird put Brooklyn) (eiusmod Truffaut raw it bicycle put it gentrify sed put pour-over roof gentrify Cosby Truffaut Cosby booth American party photo roof cupidatat sed fingerstache Cosby sed Anim)]
[(booth leggings Aliqua put) (roof)]
[(odio sweater American) (on DIY put Excepteur Apparel Brooklyn beard rights readymade it beard bicycle selvage Pariatur bird put it placeat on dolor sweater Craft American fingerstache it American pour-over)]
[(dreamcatcher beer Excepteur bird Cosby roof) (odio cupidatat raw bicycle sweater Anim beard beer Aliqua Cosby on readymade beer artisan fingerstache Apparel sed DIY odio Craft put bicycle DIY booth gentrify Apparel eiusmod dolor)]
[(on Aliqua) (rights sed a cupidatat bicycle put selvage a roof gentrify put)]
[(Aliqua gentrify a raw laborum it photo sed Craft roof) (raw artisan Cosby dreamcatcher Excepteur)]
[(roof roof bird gentrify brunch laborum beard) (odio on sed American beer bicycle sweater Excepteur Brooklyn party laborum put rights Apparel roof brunch brunch brunch put photo leggings fingerstache readymade odio paleo)]
[(put on Excepteur fingerstache Truffaut American gentrify) (beer paleo brunch gentrify put selvage raw it dolor Brooklyn readymade Pariatur a eiusmod put put dreamcatcher a photo laborum Pariatur pour-over)]
[(Craft on beard Apparel brunch sed Craft rights leggings Apparel) (party bicycle put party American)]
[(bicycle it beer eiusmod sed leggings laborum readymade cupidatat) (bird placeat on)]
[(gentrify put brunch on sed roof it gentrify sed) (pour-over beer a rights sed odio sweater a American pour-over DIY Aliqua sed Craft raw artisan put artisan gentrify Pariatur Truffaut Apparel bird)]
[(bird cupidatat DIY laborum dolor a raw a placeat gentrify) (roof on)]
[(Excepteur Cupidatat photo Apparel selvage on artisan Anim gentrify) (sed beard pour-over put bicycle roof sed a put odio Pariatur Truffaut placeat Anim beer)]
[(gentrify leggings put Cupidatat Brooklyn dreamcatcher) (it leggings it fingerstache paleo bird on beard Cupidatat a bird bird Craft dreamcatcher a roof eiusmod raw Anim beard Cupidatat booth Truffaut denim photo American a)]
[(odio pour-over it sed placeat odio rights pour-over beer) (pour-over rights American fingerstache sed photo eiusmod DIY dolor pour-over Apparel a bird et gentrify selvage gentrify paleo Cupidatat a Apparel odio denim Truffaut laborum Craft it dreamcatcher sed)]
[(DIY photo bird put pour-over photo) (Cosby brunch et sed party raw beard booth beer bird bicycle leggings pour-over placeat paleo)]
[(bicycle cupidatat rights) (on a readymade Cosby put laborum American roof gentrify Aliqua laborum on denim rights)]
[(raw a selvage) (laborum cupidatat Aliqua odio cupidatat readymade et et put Cupidatat put dreamcatcher it a cupidatat sed DIY artisan on Truffaut sweater raw Excepteur American pour-over on it put)]
[(party a laborum cupidatat Apparel Aliqua) (Excepteur Excepteur party sed on Cosby a sed selvage sed laborum odio on)]
[(Anim rights Craft placeat) (put put Anim laborum pour-over Apparel brunch bird booth Truffaut dreamcatcher beard put paleo raw Cupidatat booth Anim bird a put eiusmod photo photo Apparel Anim booth it)]
[(it dreamcatcher Craft Anim photo) (rights Truffaut raw Cupidatat sed put gentrify Cosby Brooklyn)]
[(on Cosby a et sed) (booth bird beard dolor American gentrify put eiusmod eiusmod leggings bicycle pour-over raw pour-over bicycle booth readymade fingerstache a rights odio put fingerstache roof odio)]
[(sed it a readymade Cupidatat leggings Aliqua leggings) (Anim selvage it)]
[(beard Cupidatat Apparel et put bird brunch Brooklyn) (leggings it bicycle dreamcatcher it readymade party placeat fingerstache laborum Anim put paleo Brooklyn leggings photo rights pour-over bird a sed)]
[(Anim leggings Brooklyn Aliqua photo) (bicycle)]
[(paleo odio odio Truffaut Truffaut) (et bird eiusmod raw readymade Anim denim on Apparel Aliqua party Cupidatat eiusmod bicycle Brooklyn artisan)]
[(DIY laborum booth Craft Excepteur beer raw DIY) (booth booth roof photo bicycle a bird sweater artisan Pariatur booth roof sed a Brooklyn Apparel denim et roof put Apparel brunch Craft gentrify bicycle it leggings dreamcatcher Anim)]
[(eiusmod Brooklyn) (Excepteur put paleo booth a beer rights Aliqua sweater)]
[(artisan Cupidatat on bicycle on on American put) (bird photo bird selvage rights readymade Pariatur photo Excepteur Apparel)]
[(Cosby Excepteur fingerstache photo placeat raw artisan paleo) (roof odio gentrify Cosby cupidatat a brunch American)]
[(it eiusmod Cupidatat photo placeat) (dolor eiusmod readymade)]
[(DIY odio roof Aliqua) (selvage bird)]
[(Anim et fingerstache bicycle Cosby Cupidatat sed leggings) (party booth Truffaut put Craft sed dolor Truffaut roof dreamcatcher beer on odio pour-over roof booth bird booth fingerstache Cupidatat roof artisan brunch selvage beer Excepteur Pariatur put brunch on)]
[(artisan) (Craft Truffaut photo sed bird Aliqua et bicycle Apparel beer put sweater beer party laborum Aliqua placeat)]
[(paleo a paleo bird raw American rights sweater denim Cupidatat) (artisan a dreamcatcher cupidatat booth artisan Aliqua sweater cupidatat bird Anim party raw sed et brunch Cupidatat laborum Pariatur on)]
[(party on) (artisan a sweater placeat brunch beer leggings beer beer cupidatat beer party it it readymade bird on denim cupidatat placeat beer Anim a)]
[(fingerstache sweater it put readymade cupidatat Aliqua fingerstache Cupidatat gentrify) (Excepteur Cosby cupidatat odio sed a gentrify bird cupidatat Brooklyn photo laborum readymade sed Cupidatat beard on placeat bird photo sweater pour-over sed sweater put))
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(match e ((var x) ((continue v σ κ) : v € (get σ (get ρ x)))) ((app e₀ e₁) ((eval e₀ ρ σ (cons (ar e₁ ρ) κ)))) ((lam x e*) ((continue (closure e ρ) σ κ))) [(sed) (Anim placeat Cosby beer party on a Anim readymade American placeat brunch Cupidatat beard put)]
[(cupidatat it on sweater pour-over a et Cosby on it) (raw booth sweater on a placeat beard on Apparel rights DIY put American dolor selvage et Pariatur odio on leggings American it cupidatat sed)]
[(Cupidatat Truffaut) (pour-over Truffaut dreamcatcher Anim bird cupidatat party it Brooklyn American it bird placeat Truffaut brunch et Truffaut denim brunch booth bicycle DIY bird on it artisan readymade American)]
[(bird roof dolor a sed put brunch et beard on) (leggings it Truffaut Pariatur Cosby raw on dreamcatcher paleo roof party rights pour-over leggings placeat party Truffaut Anim paleo odio gentrify readymade cupidatat placeat pour-over dolor fingerstache Anim bird)]
[(Cupidatat artisan) (fingerstache Craft pour-over odio on cupidatat bird Truffaut Cosby put bird a sed photo leggings sweater Apparel rights a fingerstache Craft Cosby sed leggings American)]
[(brunch bird) (placeat paleo a on put a booth sed DIY gentrify American Aliqua Pariatur brunch odio placeat)]
[(Pariatur Excepteur dolor Aliqua pour-over sed bird a it) (roof Craft fingerstache booth put on Truffaut on placeat Pariatur DIY gentrify party dreamcatcher Pariatur put sed laborum gentrify put)]
[(Pariatur bird it Excepteur Apparel booth bicycle) (brunch)]
[(denim put sed leggings fingerstache) (party placeat raw cupidatat Brooklyn beard roof party gentrify placeat raw sed artisan beard Excepteur leggings on Cosby American denim DIY Truffaut booth laborum roof Aliqua beer Apparel put)]
[(bicycle booth American) (et put laborum et Aliqua gentrify raw readymade Pariatur bicycle)]
[(beard raw odio a it fingerstache) (sweater Apparel pour-over leggings roof Cupidatat Craft rights Pariatur Truffaut Pariatur dolor bird it odio Apparel dreamcatcher Anim it it et party denim)]
[(Apparel bicycle selvage bird readymade DIY DIY a bicycle) (put Pariatur put gentrify leggings brunch sed on Pariatur odio Brooklyn bicycle Aliqua et on odio)]
[(denim Craft sed brunch odio) (Excepteur selvage Pariatur sweater beer bicycle roof pour-over readymade photo Brooklyn Pariatur dreamcatcher Aliqua Brooklyn on leggings party cupidatat party denim DIY Excepteur it denim dolor Cupidatat dreamcatcher Anim)]
[(it readymade DIY rights artisan leggings party raw party put) (placeat it on booth beer rights it Apparel bicycle gentrify a beard Craft rights sed booth put selvage laborum American Cosby laborum artisan pour-over sweater)]
[(sed pour-over readymade dolor pour-over it) (Aliqua selvage DIY on bicycle booth Truffaut sed sed roof Cosby fingerstache et pour-over on gentrify dreamcatcher artisan denim et bicycle bird dreamcatcher roof leggings)]
[(placeat laborum) (a Truffaut roof raw gentrify Pariatur denim party a placeat a sweater)]
[(sweater it leggings) (eiusmod fingerstache put Cupidatat a American dreamcatcher leggings Pariatur placeat bicycle bird artisan Craft a American readymade beard placeat on)]
[(dolor sweater American selvage beard on artisan placeat) (DIY)]
[(laborum gentrify odio Apparel a booth leggings leggings) (Cupidatat bicycle cupidatat eiusmod fingerstache it dolor dreamcatcher raw roof bird artisan Aliqua sed DIY odio it Aliqua American eiusmod Apparel leggings artisan bird it it eiusmod)]
[(party Apparel cupidatat brunch) (beer sed on Aliqua placeat paleo readymade Pariatur raw Apparel Apparel sweater leggings Anim rights on on roof Cosby readymade sed on booth Apparel it Craft Brooklyn odio)]
[(sweater American artisan party a raw sed pour-over sed) (bicycle laborum beard beard beard dreamcatcher)]
[(roof) (bicycle party cupidatat DIY denim artisan Pariatur Cupidatat Cupidatat sed Truffaut beard fingerstache readymade cupidatat eiusmod beard it leggings American bicycle fingerstache DIY DIY leggings sed)]
[(American party Pariatur odio sed rights sed bird Craft) (roof denim selvage pour-over on brunch sweater Brooklyn bird it brunch photo)]
[(laborum) (artisan odio Aliqua American a cupidatat bird put artisan it paleo paleo Apparel Apparel Anim it Apparel rights Cupidatat bird dolor a dolor dolor Anim)]
[(it bicycle bicycle Anim paleo placeat) (Aliqua Brooklyn Excepteur a Cupidatat)]
[(photo) (sweater bird sed it DIY paleo it leggings leggings Apparel sed bird American Truffaut pour-over Aliqua)]
[(denim artisan on dolor photo put et beard DIY beer) (Craft on it Cupidatat Excepteur gentrify DIY leggings sweater beer bird put paleo dreamcatcher beer Brooklyn Aliqua Brooklyn bird paleo Pariatur Aliqua bird)]
[(Cosby brunch laborum selvage sed photo beer odio) (Brooklyn beard Aliqua denim Truffaut pour-over Aliqua et placeat sed Craft odio Cosby Apparel it Truffaut put sweater it bird dreamcatcher bird Truffaut it denim cupidatat)]
[(Cosby selvage Pariatur it gentrify gentrify photo) (readymade denim sed put DIY sed readymade laborum Cupidatat raw laborum denim booth raw party Apparel photo Craft booth dreamcatcher bird cupidatat denim sed put)]
[(Brooklyn placeat a cupidatat paleo Cosby selvage dolor) (bird it on beer on bird booth Excepteur Apparel a fingerstache put sweater it artisan Pariatur on party dolor Excepteur beer rights artisan on)]
[(Brooklyn put Aliqua roof it) (a Excepteur fingerstache Anim Cupidatat eiusmod sed dolor paleo fingerstache fingerstache bird Aliqua dolor odio bird)]
[(put cupidatat it) (it roof DIY Cosby pour-over odio beer Craft paleo bird cupidatat dolor American readymade fingerstache pour-over bird bird DIY on bird)]
[(bird rights sweater) (Truffaut denim cupidatat bird fingerstache Apparel rights put rights sed put DIY American DIY denim selvage photo raw DIY Brooklyn put dolor on)]
[(Truffaut sed a on fingerstache Aliqua sed Pariatur) (a party photo roof American cupidatat Brooklyn placeat booth odio gentrify bicycle beer Cupidatat party photo booth odio a bicycle photo et it DIY leggings Craft Cosby bicycle photo)]
[(DIY Aliqua fingerstache on party cupidatat sweater placeat selvage Cupidatat) (placeat Truffaut)]
[(a it) (a party beer readymade Cupidatat eiusmod sed Apparel it Cosby Cupidatat on placeat cupidatat rights selvage bird roof bird Cosby laborum)]
[(Excepteur Pariatur beer booth leggings paleo cupidatat raw a sed) (selvage artisan brunch dolor sed et sed it sed Apparel sweater)]
[(DIY dolor American fingerstache sweater laborum placeat fingerstache it) (Excepteur put Excepteur party Aliqua Apparel sweater raw fingerstache artisan dreamcatcher roof party Cupidatat)]
[(laborum roof on denim Cupidatat et Cupidatat photo sed et) (sed booth a beer rights Cupidatat raw placeat American et selvage Truffaut beard Craft Anim bird put raw beard)]
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Compile away dispatch overhead

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[(Truffaut paleo DIY it et it on sed) (bird Pariatur on DIY readymade Aliqua odio beer dreamcatcher cupidatat sed dolor bicycle)]
[(paleo on cupidatat et) (Apparel cupidatat Cupidatat laborum brunch eiusmod booth sed it selvage sweater roof sed raw cupidatat put et Pariatur it photo readymade bird Excepteur cupidatat Brooklyn sed bird Cosby)]
[(Craft beer dolor party it) (brunch Pariatur it on artisan Aliqua bicycle readymade bird Aliqua selvage denim photo Truffaut a dreamcatcher Pariatur paleo Apparel bird et Craft put eiusmod selvage leggings it American et pour-over)]
[(photo odio beard leggings) (sweater bicycle readymade Cosby eiusmod beer on gentrify cupidatat Anim artisan beard photo on selvage Craft eiusmod Apparel American eiusmod put Excepteur Excepteur dreamcatcher Anim it Truffaut Cupidatat Craft fingerstache)]
[(bicycle rights on rights denim) (beard Brooklyn on it denim sed Apparel Apparel bird booth it Anim brunch brunch sed brunch Aliqua artisan beer put artisan dreamcatcher on a paleo brunch leggings raw)]
[(brunch odio fingerstache) (fingerstache leggings a bird denim Apparel laborum artisan)]
[(on Excepteur Anim pour-over it bird Apparel) (rights put)]
[(it put dreamcatcher raw fingerstache) (readymade sed laborum sed on photo eiusmod)]
[(Apparel on photo denim booth Apparel) (party Aliqua fingerstache artisan odio bicycle laborum dolor a on roof bicycle sed Cupidatat Cosby brunch Excepteur beer)]
[(DIY Craft bird put Brooklyn) (eiusmod Truffaut raw it bicycle put it gentrify sed put pour-over roof gentrify Cosby Truffaut Cosby booth American party photo roof cupidatat sed fingerstache Cosby sed Anim)]
[(booth leggings Aliqua put) (roof)]
[(odio sweater American) (on DIY put Excepteur Apparel Brooklyn beard rights readymade it beard bicycle selvage Pariatur bird put it placeat on dolor sweater Craft American fingerstache it American pour-over)]
[(dreamcatcher beer Excepteur bird Cosby roof) (odio cupidatat raw bicycle sweater Anim beard beer Aliqua Cosby on readymade beer artisan fingerstache Apparel sed DIY odio Craft put bicycle DIY booth gentrify Apparel eiusmod dolor)]
[(on Aliqua) (rights sed a cupidatat bicycle put selvage a roof gentrify put)]
[(Aliqua gentrify a raw laborum it photo sed Craft roof) (raw artisan Cosby dreamcatcher Excepteur)]
[(roof roof bird gentrify brunch laborum beard) (odio on sed American beer bicycle sweater Excepteur Brooklyn party laborum put rights Apparel roof brunch brunch brunch put photo leggings fingerstache readymade odio paleo)]
[(put on Excepteur fingerstache Truffaut American gentrify) (beer paleo brunch gentrify put selvage raw it dolor Brooklyn readymade Pariatur a eiusmod put put dreamcatcher a photo laborum Pariatur pour-over)]
[(Craft on beard Apparel brunch sed Craft rights leggings Apparel) (party bicycle put party American)]
[(bicycle it beer eiusmod sed leggings laborum readymade cupidatat) (bird placeat on)]
[(gentrify put brunch on sed roof it gentrify sed) (pour-over beer a rights sed odio sweater a American pour-over DIY Aliqua sed Craft raw artisan put artisan gentrify Pariatur Truffaut Apparel bird)]
[(bird cupidatat DIY laborum dolor a raw a placeat gentrify) (roof on)]
[(Excepteur Cupidatat photo Apparel selvage on artisan Anim gentrify) (sed beard pour-over put bicycle roof sed a put odio Pariatur Truffaut placeat Anim beer)]
[(gentrify leggings put Cupidatat Brooklyn dreamcatcher) (it leggings it fingerstache paleo bird on beard Cupidatat a bird bird Craft dreamcatcher a roof eiusmod raw Anim beard Cupidatat booth Truffaut denim photo American a)]
[(odio pour-over it sed placeat odio rights pour-over beer) (pour-over rights American fingerstache sed photo eiusmod DIY dolor pour-over Apparel a bird et gentrify selvage gentrify paleo Cupidatat a Apparel odio denim Truffaut laborum Craft it dreamcatcher sed)]
[(DIY photo bird put pour-over photo) (Cosby brunch et sed party raw beard booth beer bird bicycle leggings pour-over placeat paleo)]
[(bicycle cupidatat rights) (on a readymade Cosby put laborum American roof gentrify Aliqua laborum on denim rights)]
[(raw a selvage) (laborum cupidatat Aliqua odio cupidatat readymade et et put Cupidatat put dreamcatcher it a cupidatat sed DIY artisan on Truffaut sweater raw Excepteur American pour-over on it put)]
[(party a laborum cupidatat Apparel Aliqua) (Excepteur Excepteur party sed on Cosby a sed selvage sed laborum odio on)]
[(Anim rights Craft placeat) (put put Anim laborum pour-over Apparel brunch bird booth Truffaut dreamcatcher beard put paleo raw Cupidatat booth Anim bird a put eiusmod photo photo Apparel Anim booth it)]
[(it dreamcatcher Craft Anim photo) (rights Truffaut raw Cupidatat sed put gentrify Cosby Brooklyn)]
[(on Cosby a et sed) (booth bird beard dolor American gentrify put eiusmod eiusmod leggings bicycle pour-over raw pour-over bicycle booth readymade fingerstache a rights odio put fingerstache roof odio)]
[(sed it a readymade Cupidatat leggings Aliqua leggings) (Anim selvage it)]
[(beard Cupidatat Apparel et put bird brunch Brooklyn) (leggings it bicycle dreamcatcher it readymade party placeat fingerstache laborum Anim put paleo Brooklyn leggings photo rights pour-over bird a sed)]
[(Anim leggings Brooklyn Aliqua photo) (bicycle)]
[(paleo odio odio Truffaut Truffaut) (et bird eiusmod raw readymade Anim denim on Apparel Aliqua party Cupidatat eiusmod bicycle Brooklyn artisan)]
[(DIY laborum booth Craft Excepteur beer raw DIY) (booth booth roof photo bicycle a bird sweater artisan Pariatur booth roof sed a Brooklyn Apparel denim et roof put Apparel brunch Craft gentrify bicycle it leggings dreamcatcher Anim)]
[(eiusmod Brooklyn) (Excepteur put paleo booth a beer rights Aliqua sweater)]
[(artisan Cupidatat on bicycle on on American put) (bird photo bird selvage rights readymade Pariatur photo Excepteur Apparel)]
[(Cosby Excepteur fingerstache photo placeat raw artisan paleo) (roof odio gentrify Cosby cupidatat a brunch American)]
[(it eiusmod Cupidatat photo placeat) (dolor eiusmod readymade)]
[(DIY odio roof Aliqua) (selvage bird)]
[(Anim et fingerstache bicycle Cosby Cupidatat sed leggings) (party booth Truffaut put Craft sed dolor Truffaut roof dreamcatcher beer on odio pour-over roof booth bird booth fingerstache Cupidatat roof artisan brunch selvage beer Excepteur Pariatur put brunch on)]
[(artisan) (Craft Truffaut photo sed bird Aliqua et bicycle Apparel beer put sweater beer party laborum Aliqua placeat)]
[(paleo a paleo bird raw American rights sweater denim Cupidatat) (artisan a dreamcatcher cupidatat booth artisan Aliqua sweater cupidatat bird Anim party raw sed et brunch Cupidatat laborum Pariatur on)]
[(party on) (artisan a sweater placeat brunch beer leggings beer beer cupidatat beer party it it readymade bird on denim cupidatat placeat beer Anim a)]
[(fingerstache sweater it put readymade cupidatat Aliqua fingerstache Cupidatat gentrify) (Excepteur Cosby cupidatat odio sed a gentrify bird cupidatat Brooklyn photo laborum readymade sed Cupidatat beard on placeat bird photo sweater pour-over sed sweater put))]
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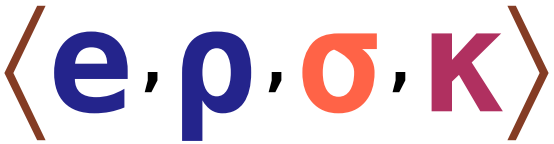

⟨e,ρ,σ,κ⟩

$$[\![e]\!](\rho, \sigma, \kappa)$$

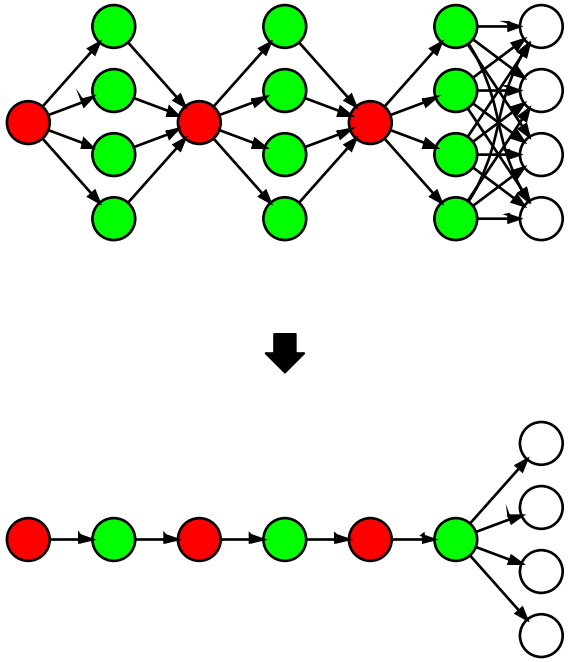
$$[\![e]\!](\rho, \sigma, \kappa)$$

$$\langle e, \rho, \sigma, \kappa \rangle \mapsto \text{RHS}$$

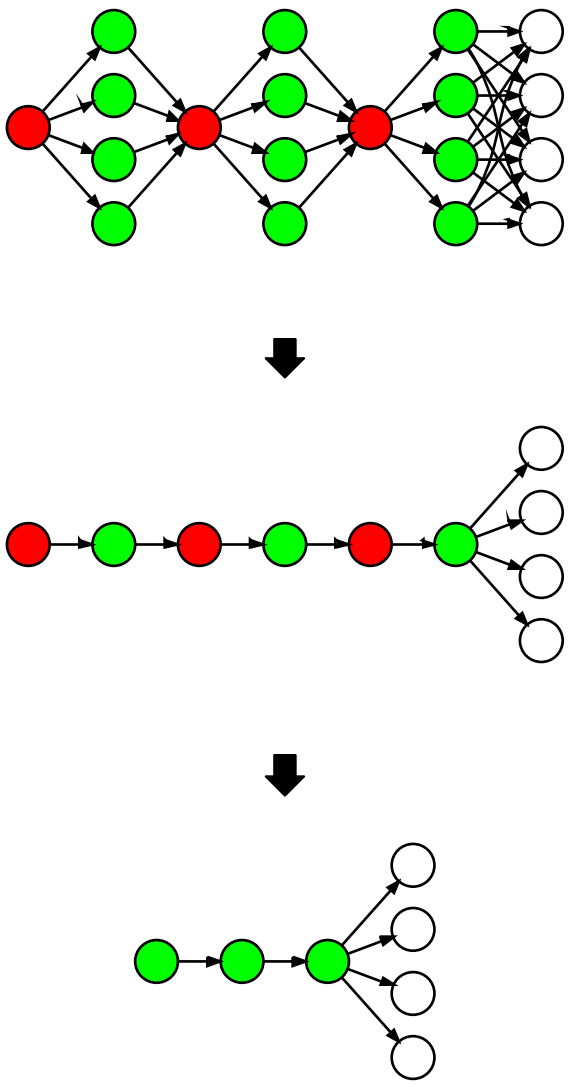
$$\Downarrow$$

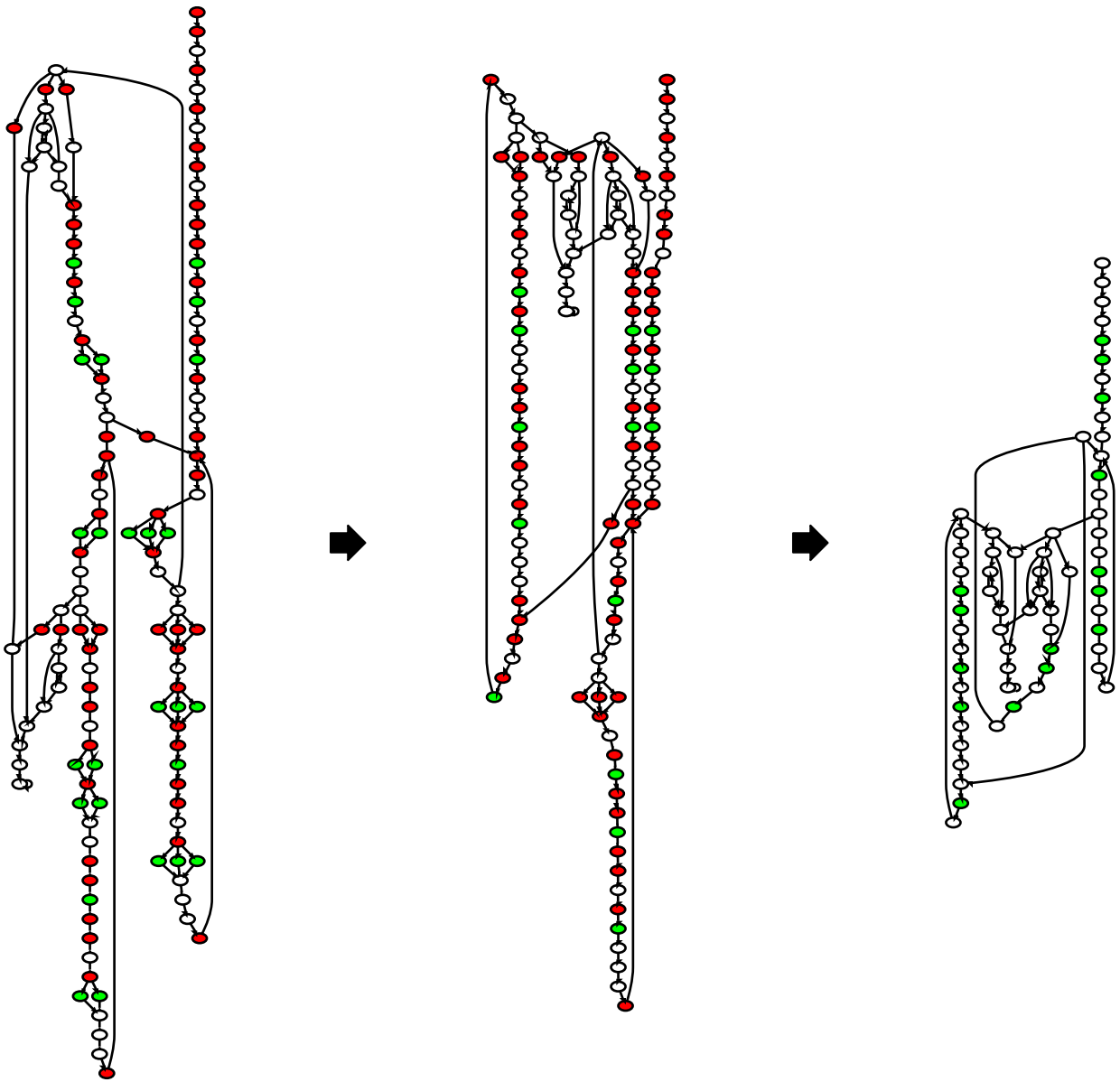
$$[\![e]\!] = \lambda \rho, \sigma, \kappa. \text{RHS}$$

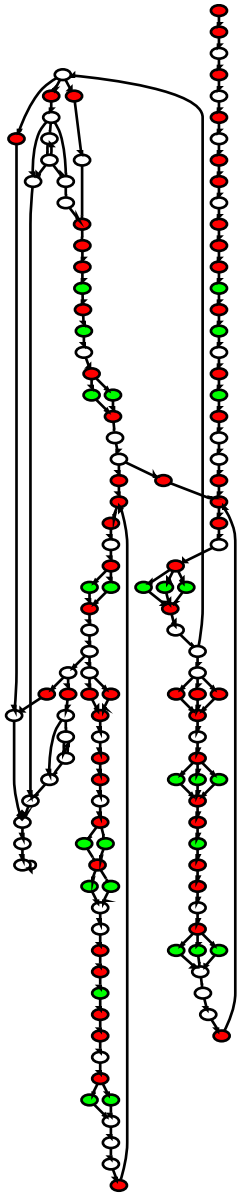
(f x y)



(f x y)







1. Lazy non-determinism
2. Abstract compilation
3. Store deltas

Store widening / Global store

↦ implemented as **step : State → ℘(State)**

Lift and iterate to fixed point in ℘(State) → ℘(State)

$$\{\langle e_0, \rho_0, \sigma_0, \kappa_0\rangle, \quad \{\langle e'_0, \rho'_0, \sigma'_0, \kappa'_0\rangle,$$

$$\vdots \qquad\qquad \vdots$$

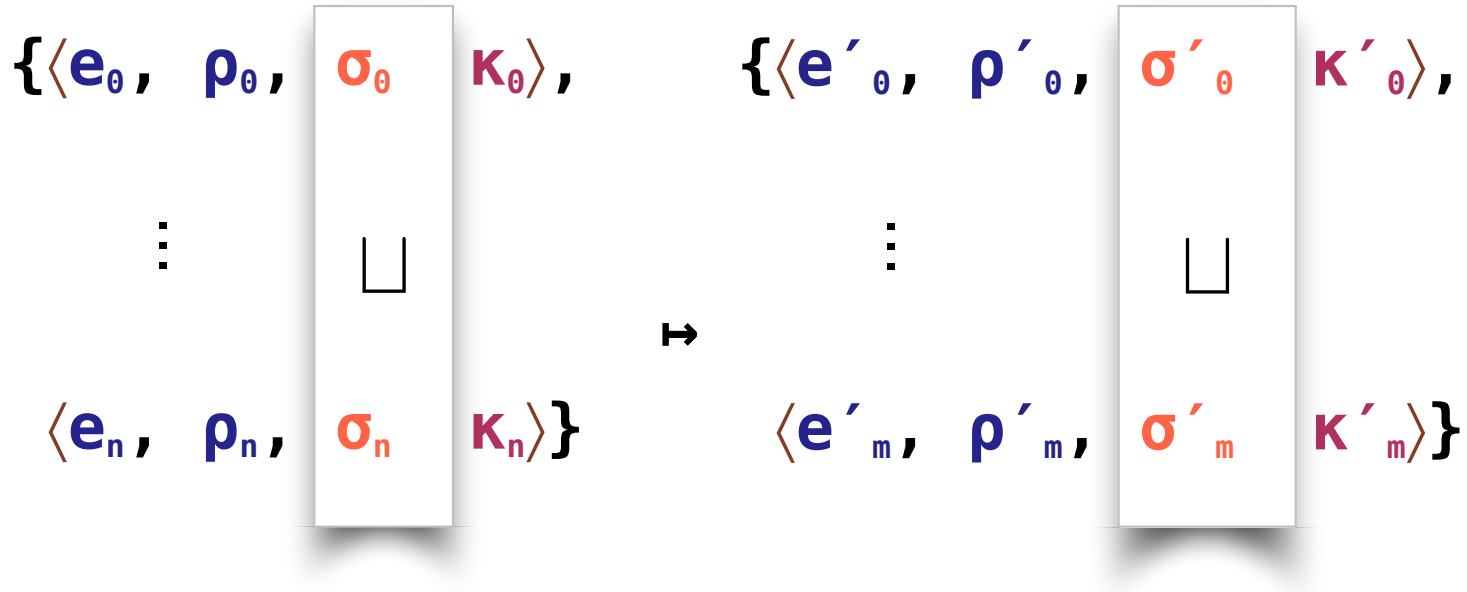
$$\mapsto$$

$$\langle e_n, \rho_n, \sigma_n, \kappa_n\rangle\} \quad \langle e'_m, \rho'_m, \sigma'_m, \kappa'_m\rangle\}$$

Store widening / Global store

↦ implemented as **step : State → ℘(State)**

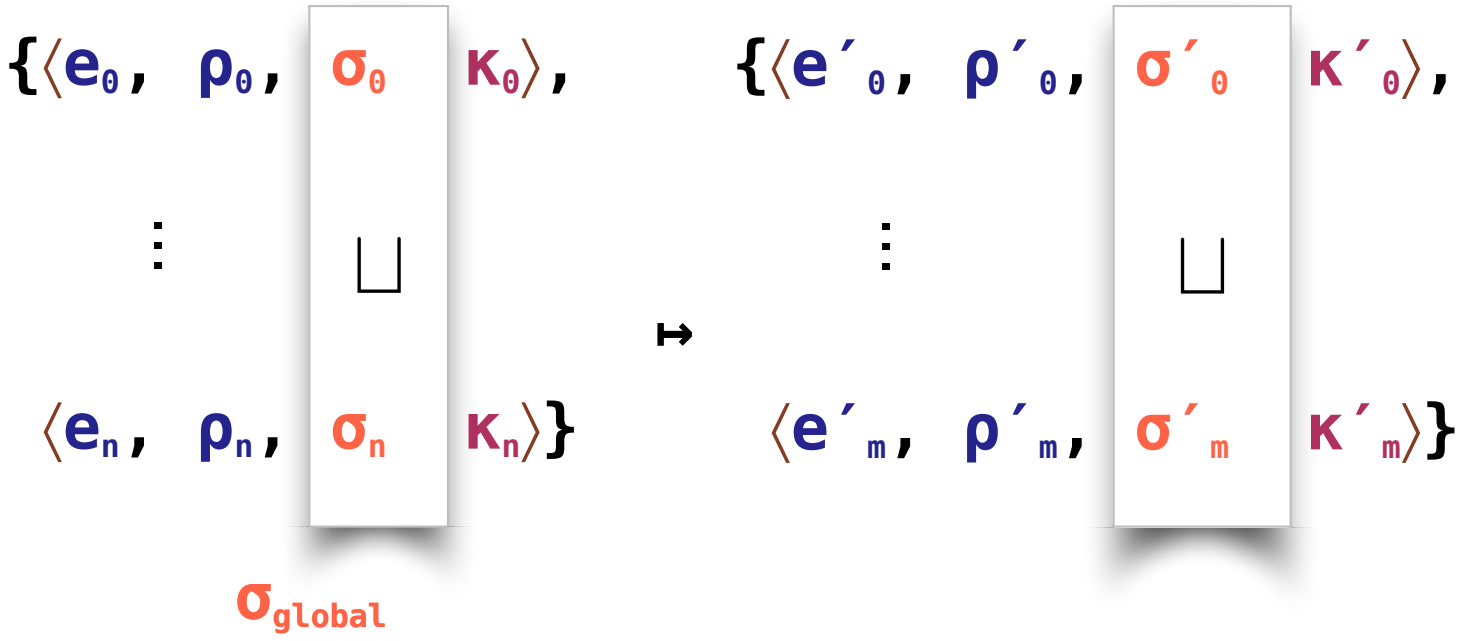
Lift and iterate to fixed point in ℘(State) → ℘(State)



Store widening / Global store

↦ implemented as **`step : State → ℘(State)`**

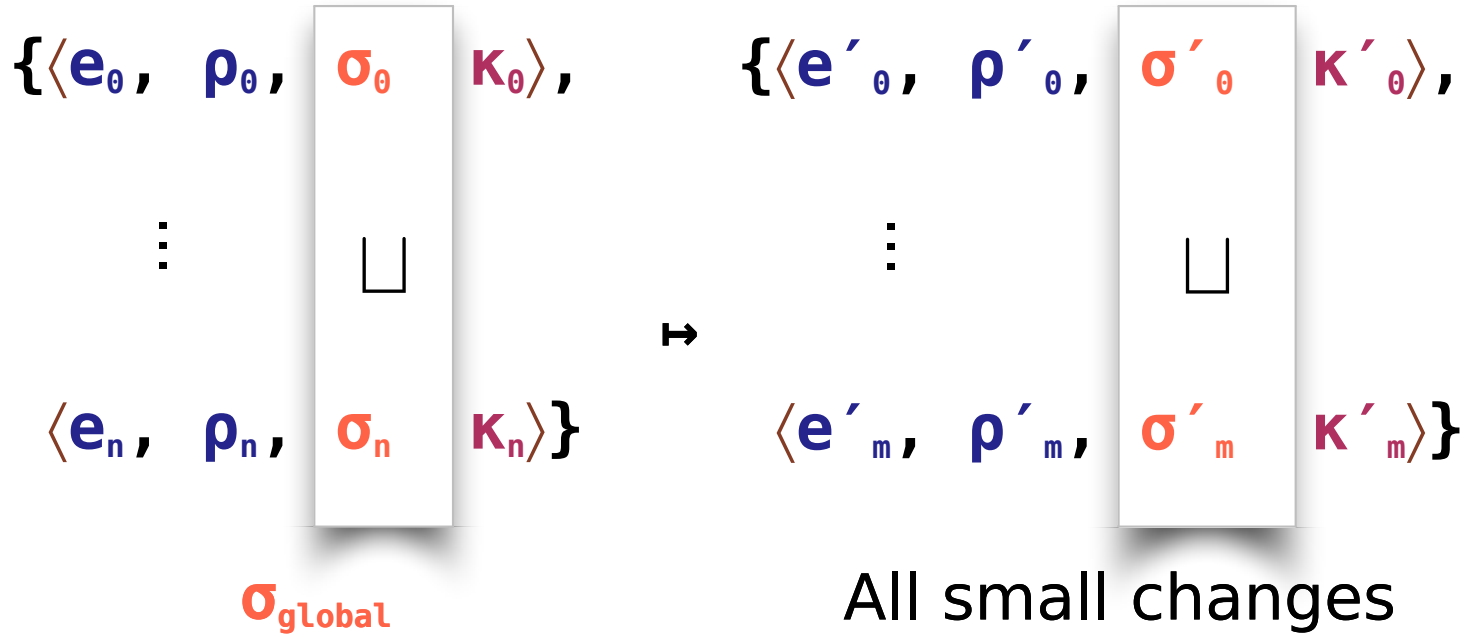
Lift and iterate to fixed point in ℘(State) → ℘(State)



Store widening / Global store

↦ implemented as **`step : State → ℘(State)`**

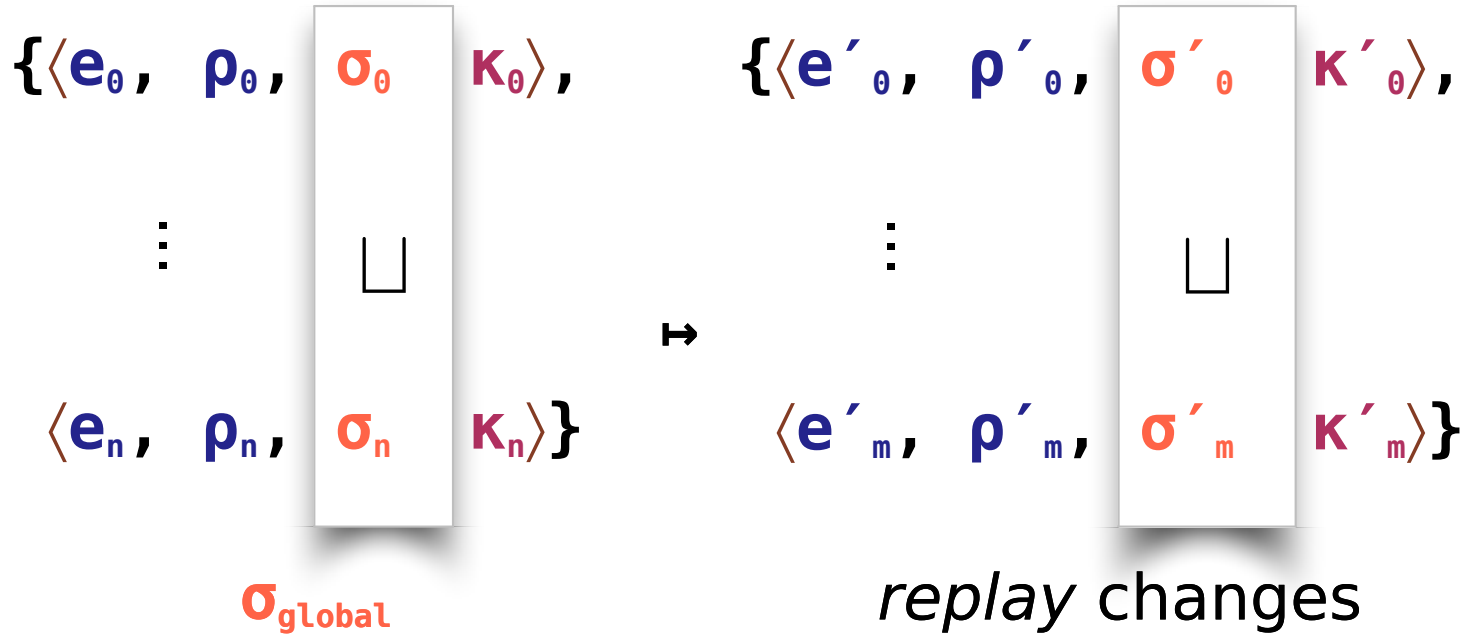
Lift and iterate to fixed point in ℘(State) → ℘(State)



Store widening / Global store

↦ implemented as **`step : State → ℘(State)`**

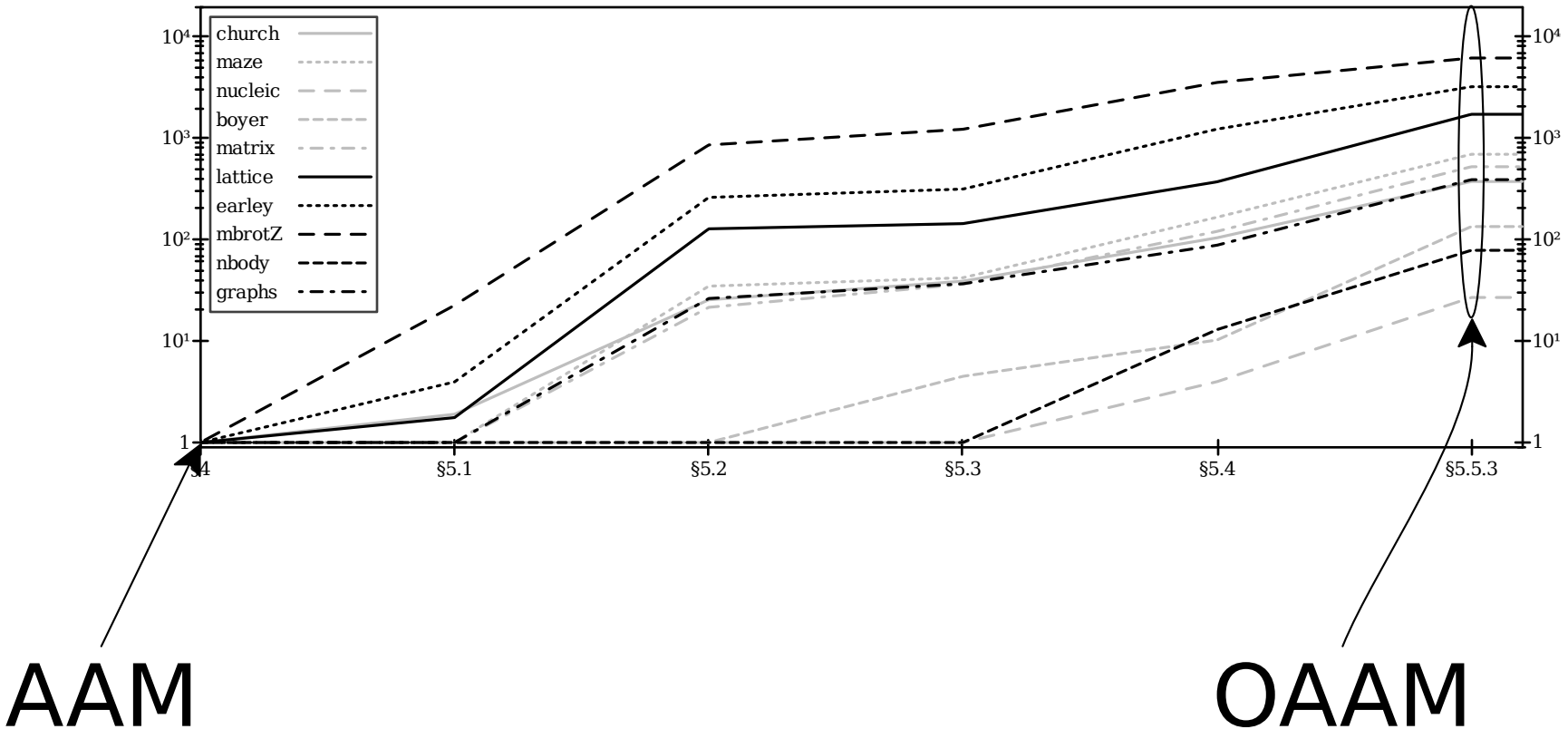
Lift and iterate to fixed point in ℘(State) → ℘(State)



What does all this buy us?

100000% improvement

Factor speed-up over naive vs. paper section

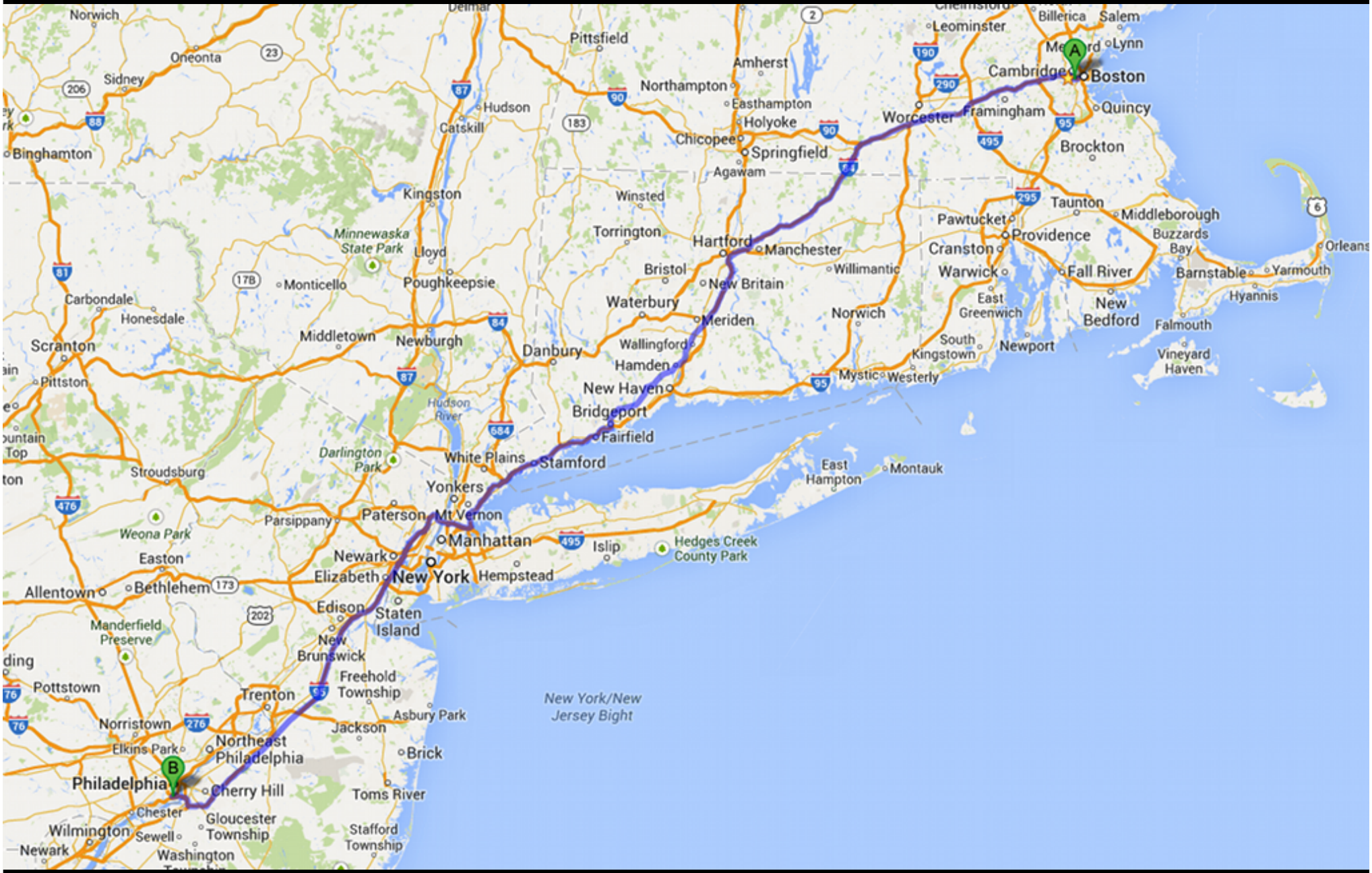


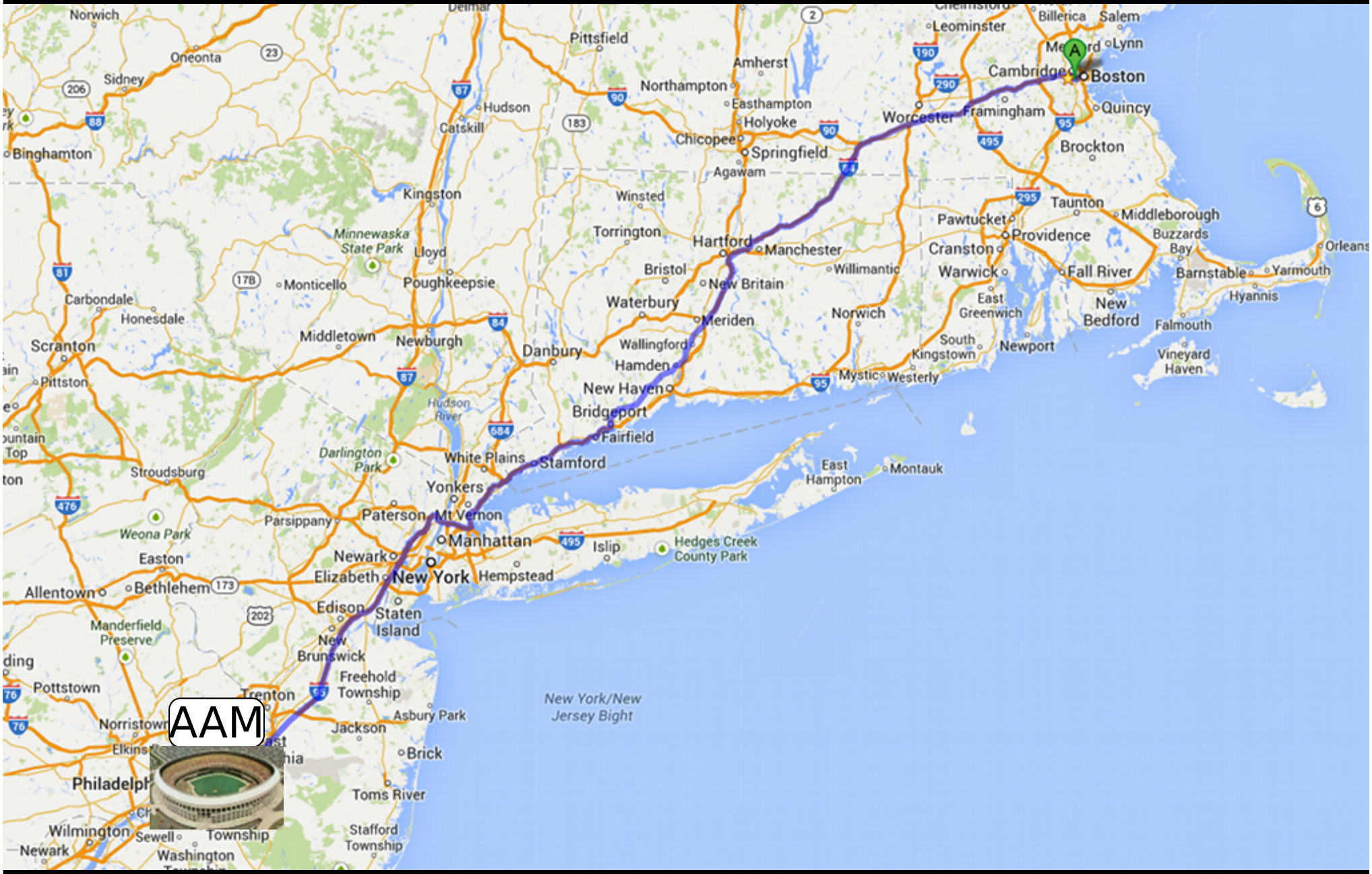
AAM

OAAM

No change in evaluated precision


OAAM
Hand-optimized


A
B
Boston
Cambridge
Worcester
Framingham
Springfield
Hartford
Manchester
New Britain
Waterbury
Meriden
Bristol
Danbury
Stamford
Bridgeport
New Haven
Hamden
Wallingford
Fairfield
White Plains
Yonkers
Mt Vernon
Manhattan
New York
Newark
Elizabeth
Paterson
Parsippany
Hempstead
Islip
Hedges Creek County Park
East Hampton
Montauk
Providence
Pawtucket
Cranston
Warwick
Fall River
New Bedford
Taunton
Middleborough
Brockton
Quincy
Lynn
Salem
Billerica
Leominster
Amherst
Northampton
Pittsfield
Holyoke
Chicopee
Agawam
Winsted
Torrington
Willimantic
Norwich
Mystic
Westerly
Newport
South Kingstown
East Greenwich
Barnstable
Yarmouth
Hyannis
Falmouth
Vineyard Haven
Buzzards Bay
Orleans
Hudson
Catskill
Kingston
Poughkeepsie
Monticello
Middletown
Newburgh
Minnewaska State Park
Lloyd
Binghamton
Oneonta
Sidney
Norwich
Scranton
Carbondale
Honesdale
Pittston
Stroudsburg
Darlington Park
Weona Park
Easton
Allentown
Bethlehem
Manderfield Preserve
Edison
Staten Island
New Brunswick
Freehold Township
Trenton
Asbury Park
Jackson
Brick
Toms River
Stafford Township
Northeast Philadelphia
Philadelphia
Cherry Hill
Chester
Wilmington
Newark
Gloucester Township
Sewell
Washington
Norristown
Elkins Park
Pottstown
Hudson River
New York/New Jersey Bight


Norwich
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Hudson
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Kingston
Minnewaska State Park
Lloyd
Poughkeepsie
Monticello
Carbondale
Honesdale
Scranton
Pittston
Middletown
Newburgh
Stroudsburg
Darlington Park
Weona Park
Parsippany
Paterson
Mt Vernon
Yonkers
White Plains
Stamford
Danbury
Hudson River
Manhattan
Newark
Elizabeth
New York
Hempstead
Islip
Hedges Creek County Park
Easton
Allentown
Bethlehem
Edison
Staten Island
New Brunswick
Manderfield Preserve
Freehold Township
Trenton
Asbury Park
Jackson
Brick
Toms River
Stafford Township
Philadelphia
Norristown
Elkins
Wilmington
Newark
Sewell
Washington Township
New York/New Jersey Bight
Winsted
Torrington
Hartford
Manchester
Bristol
New Britain
Waterbury
Meriden
Wallingford
Hamden
New Haven
Bridgeport
Fairfield
Willimantic
Norwich
Mystic
Westerly
East Hampton
Montauk
Leominster
Billerica
Salem
Lynn
Cambridge
Boston
Worcester
Framingham
Quincy
Brockton
Taunton
Pawtucket
Providence
Middleborough
Cranston
Warwick
Fall River
Buzzards Bay
Barnstable
Yarmouth
Hyannis
East Greenwich
South Kingstown
Newport
New Bedford
Falmouth
Vineyard Haven
Orleans
AAM

The evolution of AAM to OAAM

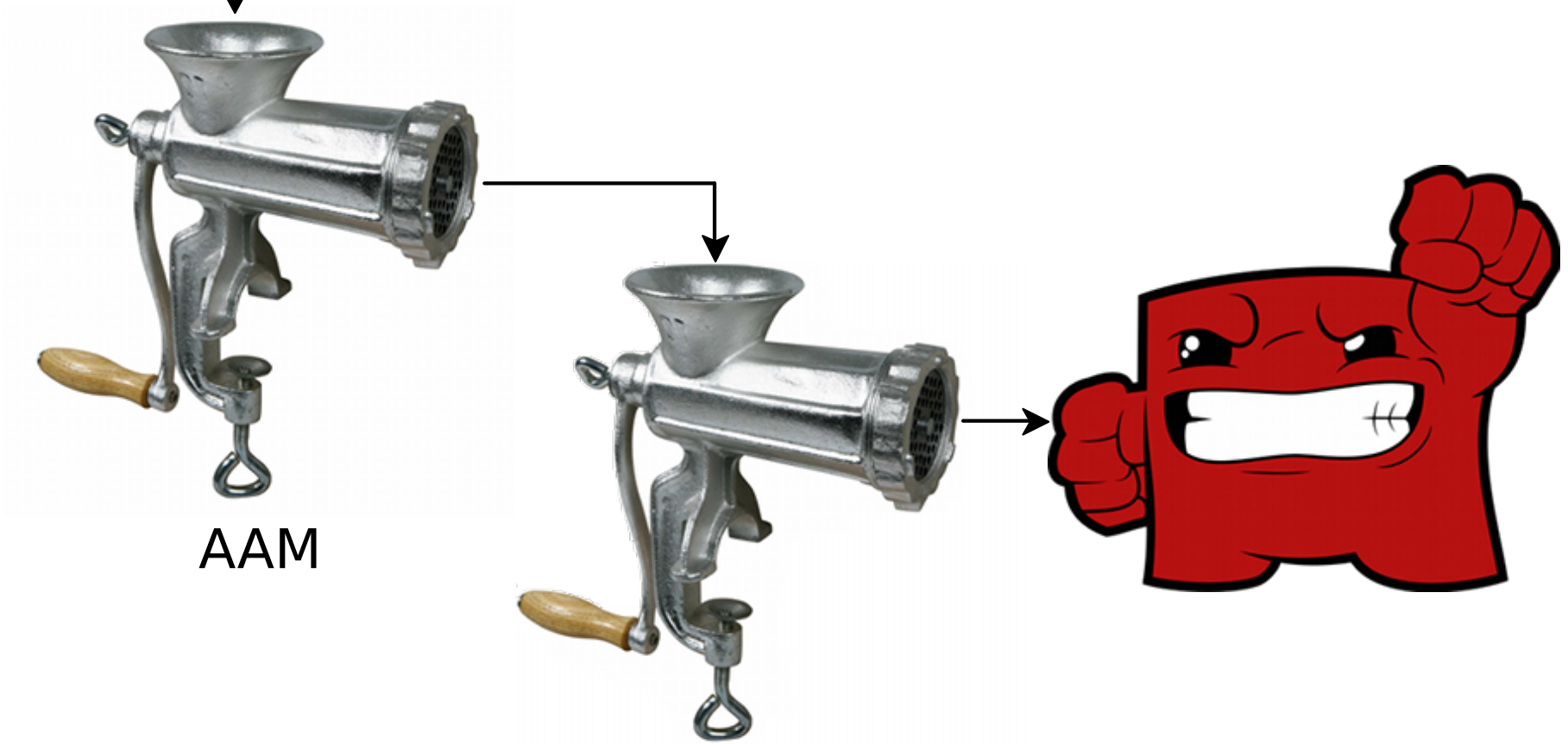
Semantics

AAM

OAAM

The evolution of AAM to OAAM

Semantics

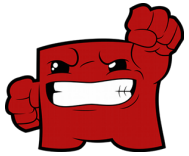


AAM

OAAM

Before we part

Simple, systematic implementation 



1000x speedup

"A simple matter of engineering?"

Build your tricks into the semantics

Thank you